Analysis of the Problem of Electric Buses Charging in Urban Transport

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Abstract. The electric bus is an ecological means of transport. Its operation reduces the negative impact on the environment. The transition to electric urban transport is in line with the concept of sustainable development of the European Union and the Polish “Act on Electromobility and Alternative Fuels”. Today, on the streets of Polish cities, electric buses are increasingly replacing diesel-powered vehicles. The electrification of the bus fleet requires the establishment of an appropriate infrastructure for charging vehicle batteries. Electric chargers are placed at the end stops of the communication lines. When the bus stops at the end of the route, it is possible to partially recharge the energy consumed. The amount of recharged energy depends on many factors, such as the power of the charger, bus stop time, possible waiting time for a free charger, etc. In the event of incomplete replenishment of the consumed energy, the completion of subsequent courses may be endangered.

The article proposes an approach to the analysis of the process of charging electric buses at the end stops of the route, taking into account the characteristics of the vehicle battery, the intensity of vehicle traffic, the characteristics of stationary chargers and its quantity. The proposed approach uses queue theory to describe the process of bus charging. The result of the research is the estimation of the use of the chargers in different configurations and the estimation of the state of charge of the bus battery for selected timetables.

Keywords: electric bus, battery charging, electromobility, queue theory, public transport


Анализ проблемы при зарядке электрических автобусов городского транспорта

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Реферат. Электрический автобус является экологическим средством передвижения, эксплуатация которого снижает негативное воздействие на окружающую среду. Переход на городской электрический транспорт соответствует концепции сбалансированного развития Евросоюза и Закону об электромобиле и альтернативных видах топлива, принятому в Польше. Сегодня на улицах польских городов все чаще можно увидеть электрические автобусы, которые заменяют дизельные транспортные средства. Электрификация автобусного парка требует создания соответствующей инфраструктуры для зарядки аккумуляторных батарей автобусов путем размещения электрических зарядных устройств на конечных остановках маршрутов. По прибытии на конечный пункт, оборудованный зарядным устройством, автобус может частично восполнить энергию, потраченную на преодоление маршрута. В случае неполной подзарядки батареи автобус может частично восполнить энергию, потраченную на преодоление маршрута. Ее объем зависит от многих факторов, таких как: мощность зарядного устройства, интервал времени перед началом следующего курса, потенциальное ожидание в очереди к зарядному устройству и т. п. В случае неполной подзарядки батареи автобус может частично восполнить энергию, потраченную на преодоление маршрута. Ее объем зависит от многих факторов, таких как: мощность зарядного устройства, интервал времени перед началом следующего курса, потенциальное ожидание в очереди к зарядному устройству и т. п. В случае неполной подзарядки батареи завершение
**Introduction**

Electromobility is a contemporary trend in the development of transport systems in Europe [1]. The concept of sustainable development of the European Union assumes, among other things, the transition of public transport to electric vehicles. In accordance with “A European Strategy for Low-Emission Mobility” [2], the development of transport using alternative fuels, including electricity, is a priority for the coming years. In Poland, the development of ecological means of transport is promoted by the “Act on Electromobility and Alternative Fuels” approved by the government [3].

Electric buses are one of the green means of transport. Using an electric bus as a replacement for a combustion vehicle reduces the negative impact on the environment, among others by reducing greenhouse gas emissions and noise levels. Therefore, on the streets of European cities, electric buses increasingly replace traditional combustion buses [4].

The introduction of electric buses in public transport is related to the development of vehicle charging infrastructure. During operation on the line, the bus consumes electricity. The source of this energy is a battery placed on the bus. Contemporary technologies of the lithium-ion batteries production [5] allow to reach the capacity of the battery, which is enough to perform several trips on the route without recharging the battery. Next, the battery must be recharged in order to ensure a continuous vehicle operation during the working day. It is possible to recharge the battery when the bus is scheduled to stop at the end of the route. The consumed energy is replenished by electric chargers located at the end stops of the route. The amount of energy replenished depends directly on the power of the charger, the type of battery and the charging time.

When designing timetables, various criteria are considered, including route length, number of stops, intensity of passenger flows, number of vehicles, minimization of transfer times, etc. Optimally designed public transport timetable requires to agree on passengers’ expectations and carrier’s capabilities. Switching urban bus transport to electric vehicles introduces additional restrictions to the timetable planning system, which are related to the charging time of the electric vehicle. If the bus does not replenish all the energy used for the course while staying at the end of the line, the next course may be threatened by insufficient battery state. The situation becomes more complex if chargers are shared between buses with different lines at common end stops.

Therefore, the use of electric buses instead diesel buses in urban transport requires a preliminary analysis of existing timetables in order to estimate the energy consumption of the bus along the route, the quantity energy supplemented during stopovers depending on the number and characteristics of chargers, and the rules of management of the buses charging process at the routes end stops.

The paper proposes and analyses a model of a system for managing the process of charging electric buses with the use of the queuing theory. The purpose of the model is to estimate the conditions needed for the handling of an urban transport line by electric buses. The input data of the model are the number and power of chargers, and the timetable of bus travel. The estimation of battery state depends on the number of trips made, the estimation of the time of using the chargers and the total power consumption at the end stop.

**Problem definition**

Let all the lines that will be operated by electric buses be represented by the set \( R = \{r_1, r_2, \ldots, r_n\} \). Each line \( r_i \in R \) can be described by the characteristics \( r_i = \{s_i, s_f, d\} \), where \( d \) – route length; \( s_i, s_f \) – starting and ending stops of the line (Fig. 1). The \( r_i \) line can start and end at the same stop, i.e. \( \exists r_i \in R \) for which \( s_i = s_f \). The same end stop may belong
to several lines of communication, i.e., \( \exists x_i \in S \) for which \( r_{a_1} \cup r_{b_1} \neq 0 \). It is assumed that each line \( r_i \in R \) is operated exclusively by electric buses (without combustion engines), which represent the non-empty finite set \( A_i = \{ a_1^i, a_2^i, ..., a_n^i \} \). All electric buses are represented by the finite set \( A = \{ A_1, A_2, ..., A_p \} \). It is also assumed that one bus serves only one line, i.e., \( \forall A_i, A_j \in A, i \neq j \implies A_i \cap A_j = \{ \} \).

The source of energy driving the electric bus is an accumulator battery with a capacity of \( E_{\text{bat}} \), kWh. The electricity used to cross the route of the \( r_i \) urban transport line is \( E_{\text{route}}(r_i) \), kWh. The value of energy consumption for each route was obtained from experimental measurements or estimated by modelling methods.

The energy of the bus battery used for the route is partially replenished when the bus stops at the end of the route. It is assumed that both ends of the route are equipped with loaders. For technical reasons, the possibility of charging the batteries at intermediate stops is excluded: due to the short stopping time and intensive traffic within the city stops, this would create additional problems that would not compensate for the effect of short-term charging. All the chargers placed at the end represent a non-empty set \( C(r_i) = \{ c_1, c_2, ..., c_l \} \).

The bus arriving at the route's end occupies the first available charger. If all the chargers are occupied now, the bus is queued up to wait for an available charger. If the estimated waiting time in the queue exceeds the scheduled waiting time at the end, the bus must leave the queue. In this case, the battery is not charged. After releasing any charger, the first bus leaves the queue and starts the charging process. The bus that has recharged battery leaves the stop. The process of bus charging is shown in Fig. 2.

**Description of the queuing system**

Let's describe the functioning of the electric buses charging system in the terminology of queuing systems. In order to classify the system, the following should be formulated: the process of arriving the buses to the end stop, the process of recharging the battery and the rules of the queue's operation. Let's the average frequency of bus arrivals at the end of the route is \( \lambda \). For example, \( \lambda = 5 \) means that in one hour at the end stop of the route arrives 12 buses. The basic factor determining the intensity of \( \lambda \) is the bus timetable. In fact, \( \lambda \) depends on the time of day: in rush hour its value will be maximum. For further calculations, the maximum \( \lambda \) values will be used.

The single \( c_i \in \{ C \} \) charger provides the bus service channel. At the end stop of the route it is possible to place any number of chargers (greater than zero). It is assumed that all chargers have identical characteristics. The \( \mu \) intensity of the bus operation process is determined by the number of \( q \) chargers and the charging time \( t_c \)

\[
\mu = \frac{q}{t_c}.
\]

When the next bus arrives to an end, it is possible that all the chargers are occupied. In this case, the bus can resign from charging or stand in a queue. The queue-free system is the simplest way to organize the bus service process. It is also friendly for the driver, who does not wait in the queue and can use this time to relax. However, it has the lowest effectiveness in processing requests. The system with unlimited queue length guarantees that the bus battery will be charged sooner or later. However, it does not ensure that the timetable is respected. In this situation, the best solution...
seems to be a queue with a limited waiting time. The bus may resign from waiting in the queue if this may delay the next trip.

Let's describe the states of the queuing system for charging buses at the stop. A set of discrete system states can be defined as $S = \{ S_0, S_1, \ldots, S_n, \ldots, S_k \}$. The initial state of the system is a state $S_0$ where all chargers are free. The occupation of the first free charger by the bus brings the system to the state of $S_1$. Let $q$ be the number of chargers at the stop. Then the states $S_0 \ldots S_{q-1}$ represents the situation when the system accepts requests and immediately serves buses that are coming to the stop. The $S_q$ state means that all chargers are occupied. In a system without a queue, the next requests will be rejected, i.e. $S_q$ is the last state of the queue-free system. In systems with a queue, the state $S_{q+1}$ represents the situation when all the chargers are busy and there is one bus waiting in the queue. The maximum number of buses in a queue is limited by the total number of buses on the route – $k$. Then the last state of the system will be $S_k$, when all the chargers are busy, and in the queue are waiting $k-q$ buses (Fig. 3).

Transitions between states occur at random time points. The system transition time from one state to the next is very small in comparison with the time where system is staying in the $S_i$. The probability of the system transition from the $S_i$ state to the different $S_j$ state does not depend on the states preceding $S_i$. For example, the system may go to the $S_1$ state (one charger is occupied), when the charging bus leaving charger (from $S_2$), or when the bus arrives to the stop and all chargers are free (from $S_0$). Therefore, the process of buses handling at the stop can be considered as Markov process [6]. We also assume that the number of buses on the route is very large ($k >> 1$) and the scheduled intervals of their courses are much smaller than the journey time (otherwise the problem of handling many buses at the end simply does not exist). In this case the number of requests in a random time interval $\tau_i$ will not depend on the number of requests in another interval $\tau_2$, provided that the intervals do not overlap: $\tau_1 \cap \tau_2 = \{ \}$. So, the process of arrival of the buses at the end stop can be considered as the Poisson’s process [7].

Let $p_i$ be the probability that the system is in the $S_i$ state. For so defined SMO it is possible to build a system of Kolmogorov equations for systems with a finite number of states. The sum of all probabilities is $\sum_{i=0}^{k} p_i = 1$. Accordingly, the probabilities of each state can be calculated as:

$$p_0 = \left( 1 + \frac{\lambda_{01}}{\mu_{10}} + \frac{\lambda_{12}\lambda_{01}}{\mu_{21}\mu_{10}} + \cdots + \frac{\lambda_{k-1,k}\lambda_{01}}{\mu_{k,k-1}\cdots\mu_{21}\mu_{10}} \right)^{-1};$$

$$p_1 = \frac{p_0\lambda_{01}}{\mu_{10}};$$

$$p_2 = \frac{p_0\lambda_{12}\lambda_{01}}{\mu_{21}\mu_{10}};$$

$$\cdots$$

$$p_k = \frac{p_0\lambda_{k-1,k}\cdots\lambda_{12}\lambda_{01}}{\mu_{k,k-1}\cdots\mu_{21}\mu_{10}}.$$ 

Assume that the waiting queue of buses works on the FIFO rule. The time that the bus stays at the end stop must not exceed the maximum value, which is determined by the timetable. The bus driver can resign from waiting in the queue if there is a risk of delaying departure for the next trip. Let $t_s$ is the average waiting time in the queue at the stop, after which the bus leaves the queue. In this case, the intensity of the stream of resignation from waiting can be defined as $v = 1/t_s$. Let’s introduce the channel load coefficient $\rho = \lambda/\mu$ and the resignation coefficients $\beta = v/\mu$. Then the probability that all chargers are available will be calculated as:

$$p_0 = \left( 1 + \frac{\rho^2}{1! 2!} + \cdots + \frac{\rho^n}{n!} + \frac{\rho^2}{n!} (n+\beta) + \frac{\rho^2}{(n+\beta)(n+2\beta)} + \cdots + \frac{\rho^2}{(n+\beta)(n+2\beta)\cdots(n+r\beta)} \right)^{-1};$$
The probability that \( i \) chargers are occupied and there is no bus waiting in the queue will be

\[ p_i = p_0 \frac{\rho^i}{i!}, \quad i \in [1..q]. \]

The probability for other states of the system, when the queue is not empty, depends also on the intensity of the resignation from waiting

\[ p_{q+i} = p_0 \frac{\rho^{q+i}}{q!(q+\beta)(q+2\beta)\ldots(q+i\beta)}, \quad i > 0. \]

The average number of queued vehicles can be calculated as \( \bar{w} = \sum_i p_{q+i} \). Then the average number of resigning buses can be estimated as \( R = \bar{w}v \).

The average number of occupied chargers will be

\[ C = \frac{A}{\mu} = \rho - \bar{w}\beta. \]

**Calculation of system characteristics using real data**

We will calculate the system characteristics using the real data obtained for chosen bus line [8]. The intensity of bus arrivals at the end stop was calculated using timetable at rush hours of the working day: \( \lambda = 14 \). The average time \( t_c \) of the bus stay at the end stop equals 10 minutes. So, the intensity of the vehicles handling process can be calculated from (1) as \( \mu = q/t_c = q/(10/60) = 6q \), where \( q \) – variable number of chargers. We assume that the average bus waiting time in a queue does not exceed the total waiting time at the end stop of the route and equals \( t_s = 10 \) min. Then the intensity of the resigning will be \( v = \frac{1}{t_s} = 6 \). The results of the calculations are presented in Tab. 1.

We will analyze obtained data. Fig. 4 shows the dependence of the probability of the system staying in the states \( p_0 \ldots p_4 \) from the charger’s number \( q \). The probability of \( p_0 \) (all chargers are free) increases with increasing \( q \). Of the remaining states of the system, it most often stays in \( p_1 \) when exactly one vehicle is in charging. The maximum value \( p_1 = 0.36 \) is reached for the number of chargers \( q = 3 \).

### Table 1

<table>
<thead>
<tr>
<th>Calculated characteristics of the system</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of chargers ( n )</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Intensity of bus arrivals at end stop ( \lambda )</td>
<td>14</td>
<td>14</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>Intensity of charging process ( \mu )</td>
<td>6</td>
<td>12</td>
<td>18</td>
<td>24</td>
</tr>
<tr>
<td>Intensity of resigning process ( v )</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>Resignation coefficient ( \beta = v/\mu )</td>
<td>1.00</td>
<td>0.50</td>
<td>0.33</td>
<td>0.25</td>
</tr>
<tr>
<td>Channel load coefficient ( \rho = \lambda/\mu )</td>
<td>2.33</td>
<td>1.17</td>
<td>0.78</td>
<td>0.58</td>
</tr>
<tr>
<td>Probability ( p_0 ) of the ( S_0 ) system state (all chargers are free)</td>
<td>0.10</td>
<td>0.30</td>
<td>0.46</td>
<td>0.56</td>
</tr>
<tr>
<td>Absolute throughput ( A ) (average number of buses charged per unit time)</td>
<td>5.42</td>
<td>11.28</td>
<td>13.76</td>
<td>13.98</td>
</tr>
<tr>
<td>Relative throughput ( Q = A/\lambda )</td>
<td>0.39</td>
<td>0.81</td>
<td>0.98</td>
<td>0.99</td>
</tr>
<tr>
<td>Average number of occupied chargers ( C = A/\mu )</td>
<td>0.90</td>
<td>1.88</td>
<td>2.29</td>
<td>2.33</td>
</tr>
<tr>
<td>Average number of queued buses ( \bar{w} )</td>
<td>1.43</td>
<td>0.23</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>Average number of resigned buses ( R )</td>
<td>8.58</td>
<td>2.72</td>
<td>0.24</td>
<td>0.01</td>
</tr>
</tbody>
</table>

|
Fig. 5 shows the average number of queued buses depending on chargers’ number. As can be seen, the addition of a second charger significantly reduces the length of the queue. With \( q = 3 \) the queue almost does not exist.

Fig. 6 compares the number of served buses with the number of resigning buses. For one charger about 9 of 14 buses per hour will resign from queue waiting. For two chargers, the absolute throughput of the system \( A_{q=2} \approx 11 \), i.e. about of 3 buses will resign from queuing. Three chargers will handle all requests.

We will analyze the process of replenishing the energy of the bus battery during a stop at the end of the route. For charging electric vehicles, fast and slow chargers can be used. In the case of public transport, where the bus stopover time is short and the battery capacity is relatively large, fast chargers of 100 to 600 kW must be used to charge the battery. Such chargers usually use the energy of existing tram or trolleybus networks [9].

Let us assume, that the battery capacity of a typical electric bus of 12 m length equal to \( C_{bat} = 200 \text{ kW} \cdot \text{h} \), and the time of its full charging \( t_{bat} = 2 \text{ h} \). The average battery energy consumption for the route was estimated using behavioral battery model [10] and amounts to \( E_{trac} = 30 \text{ kW} \cdot \text{h} = 15 \% C_{bat} \). During charging the bus complements \( E_{charge} = C_{bat} / t_{bat} t_{charge} = 200 / 2 \cdot 0.166 = 8.3 \% C_{bat} \).

The battery energy should be enough to ensure a continuous running of the electric bus on the line during the working day. We assume that the bus will depart on its first trip fully charged. For calculation the expected number of daily trips for the different number of chargers, the relative throughput of the system \( Q \) may be used. The expected battery status after \( n \) courses can be estimated as \( E_{bat}(q, n) = C_{bat} - n(E_{trac} - E_{charge} Q) \).

Tab. 2 shows the state of bus battery as a function of the charger’s quantity and journey number.

As can be concluded from the results obtained, if the buses are not charged at the route’s end, the battery will be discharged after 7 courses (Fig. 7).

Adding a single charger does not significantly change the situation: due to the low system throughput, most buses will resign from recharging. For two chargers at the end, they will be occupied through 94 % of the time. In this case, 81 % of battery charging requests will be handled and the average number of vehicle journeys will be 12.

Increasing the number of chargers to 3 will result in 98 % of requests being handled and the maximum number of journeys approaching to 15. Further increase in the number of chargers will not have noticeable effect.
The cost of installation be taken using the function, which should consider 2 or 3 chargers. The final decision may stop of the route in described situation, should be = 2 chargers.

In our case, the planned number of vehicle jou-

rney of day was 12, i.e. the expected result can be determined for each configuration of chargers. We have calculated a mass service system capacity, probability of bus re-signing and average queue length for different number of chargers. Introducing the capacity of bus battery and electricity consumption on the route, the average number of bus journeys on the line has been estimated. The obtained theoretical results can be used as a reference point in the design of real bus charging systems and as an input to a computer modeling system of the described problem.

So, when planning to handle buses at the end stop of the route in described situation, should be considered 2 or 3 chargers. The final decision may be taken using the function, which should consider the cost of installation, the cost of operation and the required number of vehicle journeys per day. In our case, the planned number of vehicle journeys per day was 12, i.e. the expected result can be reached with \( q = 2 \) chargers.

### Table 2

<table>
<thead>
<tr>
<th>Journey number ( n )</th>
<th>Number of chargers ( q )</th>
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<tbody>
<tr>
<td>0</td>
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<tr>
<td>1</td>
<td>85</td>
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<td>2</td>
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<td>4</td>
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<td>15</td>
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*Fig. 7. Estimated number of journeys*

### CONCLUSION

The article presents issues of planning the ser-

vice of charging batteries of electric buses in public transport. A queuing theory has been used to describe the system. Based on a timetable, we have determined parameters of incoming bus requests in order to recharge a battery. The efficiency of the bus service process, principles of queue operation and conditions of bus resignation from charging due to a limited waiting time have been determined. We have calculated a mass service system capacity, probability of bus resigning and average queue length for different number of chargers. Introducing the capacity of bus battery and electricity consumption on the route, the average number of bus journeys on the line has been estimated for each configuration of chargers. The obtained theoretical results can be used as a reference point in the design of real bus charging systems and as an input to a computer modeling system of the described problem.

### REFERENCES


Received: 08.10.2019
Accepted: 10.12.2019
Published online: 30.07.2020