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# Modeling of Capillary Shrinkage and Cracking in Early-Age Concrete

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**Abstract.** Scientific hypothesis on moistening shrinkage mechanism for cement stone and concrete has been assumed as a basis for the present paper. Physical ideas on a mechanism for cracks volume increment in a concrete model presented as two-level structure have been accepted as a theoretical basis for a calculation method of crack resistance during capillary shrinkage. These ideas are the following: a matrix of hardening cement stone with inclusions and emptiness of various forms (cracks) as result of influences that change an intense deformed state in a point and a volume. The following assumptions have been accepted while making a theoretical justification for a calculation method of shrinkable concrete crack resistance. Following this methodology approaches of fracture mechanics according to a generalized criterion have been applied in the paper. Concrete is considered as an elastic quasi-homogeneous two-component medium which consists of the following parts: a) constructive part: a matrix – a cement stone with structural elements of crushed stone, sand; b) destructive part: emptiness – capillaries cracks and pores (cavities with initial cracks in walls). Emptiness in a matrix and contact zones are presented by a coordinated five-level system in the form and sizes which are multiple to a diameter due to impacts while reaching critical sizes. These critical sizes make it possible to pass from one level into another one according to the following scheme: size stabilization – accumulation delocalization – critical concentration in single volume – transition to the following level. Process of cracks formation and their growth are considered as a result of non-power influences on the basis of crack theory principles from a condition that fields of deformation and tension creating schemes of a normal separation and shift occur in the top part of each crack at its level in the initial concrete volume. $K_{cij}(\tau)$ parameter as algebraic amount of critical values $K_{ij}$ in the whole system of all levels of cracks filling canonical volume up to critical concentration has been accepted as a generalized constant of property for concrete crack resistance in time, its resistance to formation, accumulation in volumes of micro-cracks and formation of trunk cracks with critical values. External temperature, moistening long influences create fields of tension in the top parts of cracks. Concrete destruction processes due to cracks are considered as generalized deformed-intensed state in some initial volume having physical features which are inherent to a composite with strength and deformative properties. It is possible to realize analytical calculations for assessment of tension and crack resistance of concrete at early age on the basis of a generalized criterion in terms of stress intensity factor due to modern experimental data on capillary pressure value (70 kPa in 180 min after concrete placing). The developed algorithm of calculation allows to consider factors influencing on capillary pressure: type of cement, modifiers and mineral additives, concrete curing conditions.

**Keywords:** capillary shrinkage, cracking, early age (plastic) concrete, stress intensity factor, capillary pressure, capillary forces, system of forces

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# Моделирование капиллярной усадки и трещинообразование бетона в раннем возрасте

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**Реферат.** За основу взята научная гипотеза о механизме влажностной усадки цементного камня и бетона. В качестве теоретической основы метода расчета трещиностойкости при капиллярной усадке приняты физические пред-

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ставления о механизме приращения объема пустотности (трещин) в модели бетона, представленной как двухуровневая структура: матрица твердеющего цементного камня с включениями и пустоты различной формы (трещины) как результат воздействий, изменяющих напряженно-деформированное состояние в точке и объеме. При теоретическом обосновании метода расчета усадочной трещиностойкости бетона с использованием подходов механики разрушения по обобщенному критерию приняты следующие допущения. Бетон рассматривается как упругая квазиоднородная двухкомпонентная среда, состоящая из: а) конструктивной части: матрицы – цементного камня со структурными элементами щебня, песка; б) деструктивной части: пустот – капилляров-трещин и пор (полостей с начальными трещинами в стенках). Пустоты в матрице и контактных зонах представлены соподчиненной пятиуровневой системой по форме и размерам, кратным диаметру, под воздействиями по достижении критических размеров, переходящие из уровня в следующий уровень по схеме: стабилизация размеров – делокализация накопления – критическая концентрация в единичном объеме – переход на следующий уровень. Процесс формирования и движения трещин рассматривается как результат несиловых воздействий на основе принципов теории трещин из условия, что в вершине каждой трещины своего уровня в каноническом объеме бетона возникают поля деформаций и напряжений, создающие схемы нормального отрыва и сдвига. В качестве обобщенной константы свойства трещиностойкости бетона во времени, его сопротивления образованию, накоплению в объемах микротрещин и формированию магистральных трещин критических величин принят параметр $K_{cij}(\tau)$ как алгебраическая сумма критических значений $K_{ij}$ во всей системе всех уровней трещин-пустот, заполняющих канонический объем до критической концентрации. Внешние температурные, влажностные длительные воздействия создают поля напряжений в вершинах пустот – трещин. Процессы разрушения бетона трещинами рассматриваются как обобщенное напряженно-деформированное состояние в некотором каноническом объеме, обладающем физическими особенностями, присущими композиту с прочностными и деформативными свойствами. Аналитические расчеты для оценки напряженного состояния и трещиностойкости бетона в раннем возрасте на основе обобщенного критерия в терминах коэффициентов интенсивности напряжений возможно реализовать благодаря современным экспериментальным данным о величине капиллярного давления (70 кПа через 180 мин после укладки). Разработанный алгоритм расчета позволяет учесть влияющие на капиллярное давление факторы: вид цемента, модификаторы и минеральные добавки, условия выдерживания бетона.

**Ключевые слова:** капиллярная усадка, трещинообразование, бетон в раннем возрасте, коэффициент интенсивности напряжений, капиллярное давление, капиллярные силы, система сил

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## Introduction. State of art

Modern technologies high performance concrete (HPC) are based on the following factors: low $W/C$ ratio (0.2–0.3), complex use of micro and nanosilicon dioxide and effective super plasticizers. At the same time composites with dense micro porous high-disperse structure cement C–S–H-gel are formed [1]. This structure is characterized by the following indicators: the volume of pores aren't higher, than 4–6 %; the amount of pores with $r \leq$ of 20 nanometers to 30 % of the total amount of pores. High strength and durability of these concrete are implemented in bridges, tunnels, modern roads, base plates, frameworks of high-rise buildings. These unique constructions are characterized by the high module of a surface of structures, that promotes influence of concrete moist deformations on the intense deformed state, formation and growth of cracks [2–5].

Traditionally deformations of concrete are investigated from the moment of its drying at early age at moist shrinkage against the background of the hydration processes, accompanied with hydration shrinkage. Moist deformations in structures with the high surface module are the reason of development of considerable tension during the initial and operational periods at moistening drying. There is an opinion [2–5], that hydration shrinkage influences less the general deformation of a high-performance cement stone in view of its micro porous dense structure.

Under the leadership of the academician E. N. Chernyshov, development of moist deformations at two options of process realization is investigated: a cement stone dehydration at the age of 1 days, when the general shrinkage consists of hydration (autogenous) shrinkage and moist shrinkage (drying shrinkage); dehydration of a “old” cement stone (age more than 1 year), when shrinkage is defined by moist shrinkage.

The scientific hypothesis of the mechanism of moist shrinkage of a cement stone and concrete, based on the analysis of modern theoretical representations and models of shrinkage [6–9] (tab. 1), is assumed as a basis.

*Table 1*

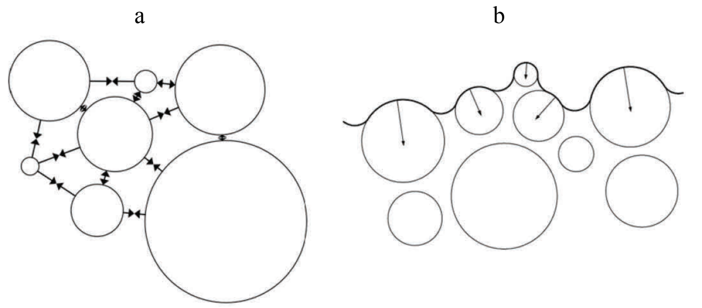
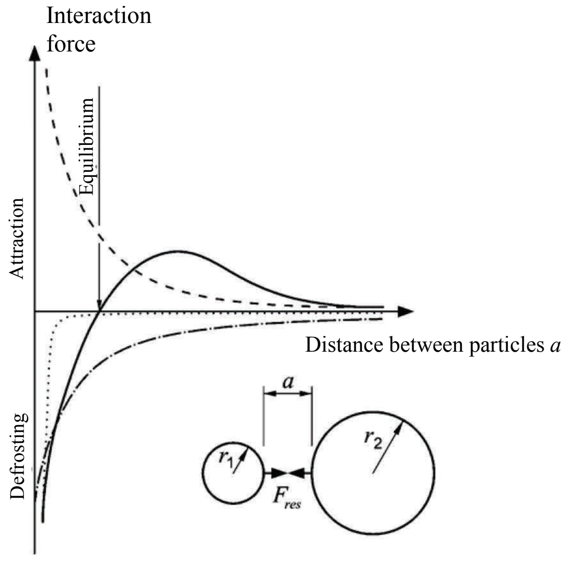
**Hypothesis of the moist shrinkage mechanism**

| Stage | Relative humidity, % | Phenomenon | Change of capillary pressure, shrinkable tension and deformations |
|---|---|---|---|
| 1 | RH = 80–95 | On the initial stage of drying, water is remove from large pores, $r$ = 100 nm | Capillary pressure is low. The size of shrinkable tension and deformations is insignificant |
| 2 | RH = 40–80 | Removal of water from a time with a radius of 20 nm < $r$ < 100 nanometers. Owing to effect of elastic restoration of a solid phase volume at reduction of comprehensive compression, expansion of system is possible | Capillary pressure increases. Moist shrinkage increases |
| 3 | RH < 40 | After removal of the capillary and connected water from pore space removal of the adsorptive and connected water begins with a surface of a solid phase, as a result its extent of compression decreases and force of elastic expansion increases | The accruing influence of forces of a superficial tension. Disappearance of forces of capillary pressure during removal of the adsorbed liquid phase. The increasing shrinkage role from intermolecular forces of interaction of particles of disperse system (rapprochement) |

At different stages of removal of water from material consistently or in parallel action of capillary forces and forces of a superficial tension, forces of internal ties in crystalline hydrates, forces of elastic counteraction of a solid phase to its deformation (tab. 2), can be shown.

*Table 2*

**The nature of influences and the influencing factors on a crack formation at capillary shrinkage**

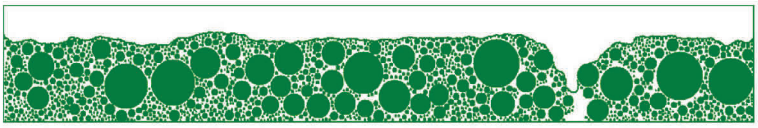
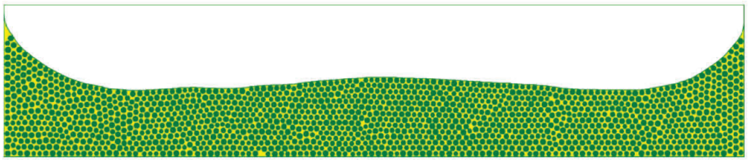
| Influence | Physics of processes. Main dependences |
|---|---|
| Impacts and the influencing factors on a crack formation at capillary shrinkage:<br>a – forces of interaction between particles;<br>b – forces which are result of capillary pressure<br>Gravitational forces aren't shown |  |
| Interaction forces<br>—— resultant force $F_{res}$<br>– – Sil Van der Vaals<br>–·– Electrostatic force<br>· · · Bourne's pushing away | Superposition  |

*Break-over, table 2*

| Influence | Physics of processes. Main dependences |
|---|---|
| 1. Van der Valls's forces<br>$A_H$ – Constant Gamakera; $r$ – particle radius;<br>$a$ – distance between particles | $F_{\nu dW} \cong A_H \frac{R}{12a^2}$ where $R = \frac{2r_1 r_2}{r_1 + r_2}$ |
| 2. Electrostatic forces<br>$\varepsilon_0$, $\varepsilon_r$ – vacuum and relative dielectric constants;<br>$\zeta$ – zet potential; $k_B$ – Boltzmann constant;<br>$T$ – absolute temperature; $e$ – elementary charge;<br>$z_+$, $n_+^{\ b}$ – valency and concentration of equivalent symmetric electrolyte | $F_{el} = -2\pi\varepsilon_0\varepsilon_r\varsigma^2 R\frac{1}{\delta}\cdot\frac{e^{a/\delta}}{1+e^{a/\delta}}$ where $\delta = \sqrt{\frac{\varepsilon_0\varepsilon_r k_B T}{2e^2 z_+^2 n_+^b}}$ |

*Table 3*

**Shrinkage modeling**

| Stages modeling | Scheme of calculation. Illustrations |
|---|---|
| 1. Suspension drying<br>• Water evaporation<br>• Formation of meniscuses between surfaces of particles<br>• Growth of capillary pressure<br>• The movement of particles under the influence of various forces<br>• Localization of deformations<br>• Formation of cracks |  |
| 2. Modelling<br>$F_i$ – the sum of forces operating on $i$ particle, including forces of internal interaction, capillary and depreciation forces of $F_d$ without gravitational forces (gravity); $g$ – acceleration of gravity | Settlement scheme<br>$F = m\ddot{u};\ \ddot{u} = \frac{F}{m} = \frac{F_1}{m} + g;\ F_d = -\alpha m\dot{u}$<br><br>$\Delta t = 2\beta\cdot\sqrt{\frac{m_{\min}}{k_{\max}}};\quad 0 \le \beta \le 1;$ $k$ – contact rigidity |

Proceeding from it, the value of moist shrinkage of material, regularity of this process are defined by force of communication of structure with water. At various stages of dehydration the balance of forces of tie of structure with water and respectively, the value of shrinkage is defined by the following criteria structural characteristics: surface area and superficial energy of a solid phase, volume amount of pores and their sizes (tab. 2). Occurring at change of a cement stone and concrete structure, change of the specified characteristics influence to force of tie of structure with water, moist shrinkage value at each stage of dehydration.

## Model of capillary pressure

In works [10–13] the model of the capillary pressure (fig. 1) is presented and experimental data of its growth (fig. 2) are presented.

The factors, influencing the capillary pressure (tab. 4) are analyzed. The main thing, the experimental data on change of capillary pressure, volume of the evaporated water and volume of a sample of concrete used as initial for calculation (fig. 3) are obtained.

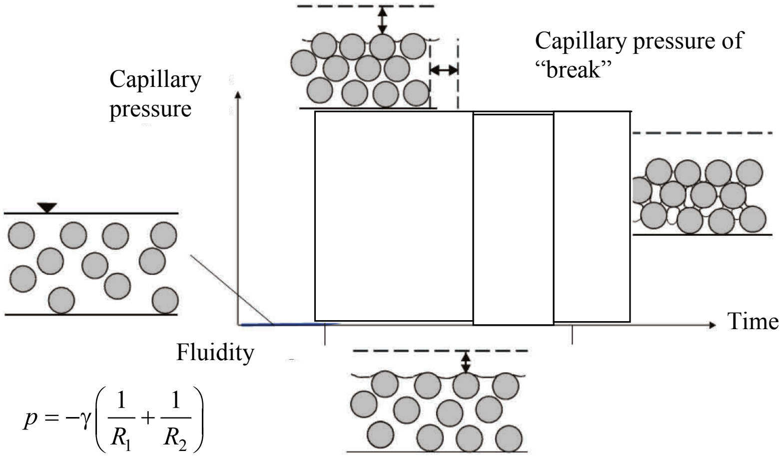


*Fig. 1.* Model of capillary pressure

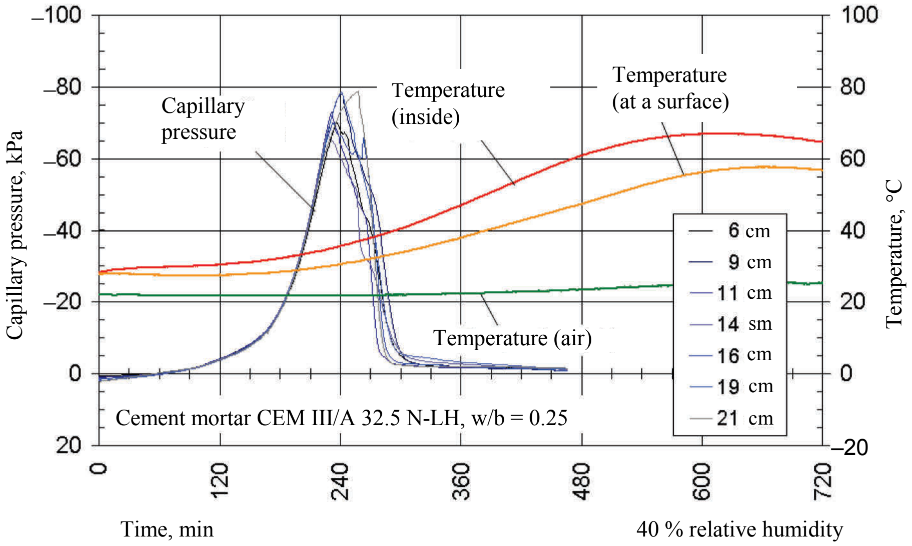
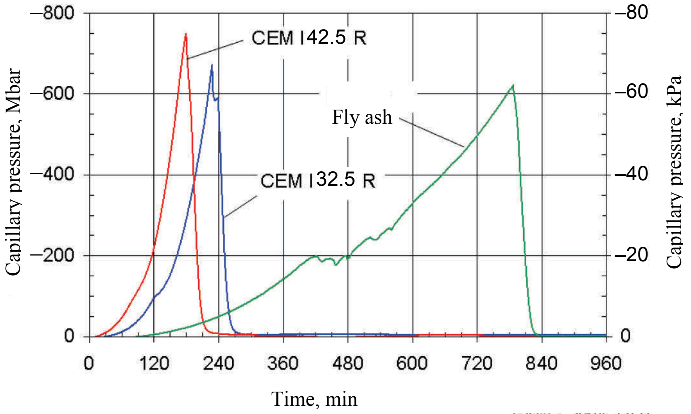


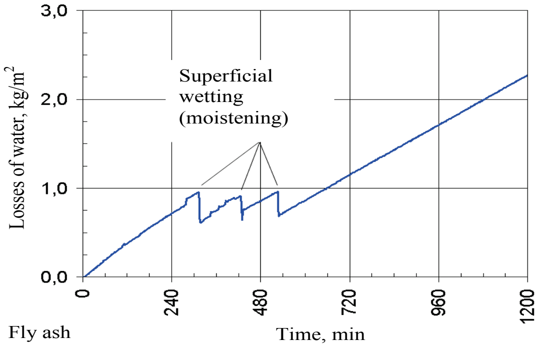
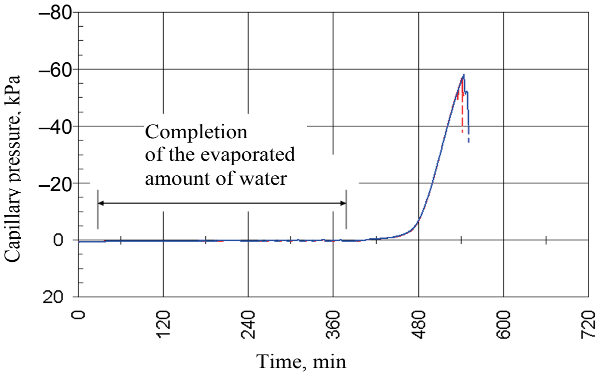
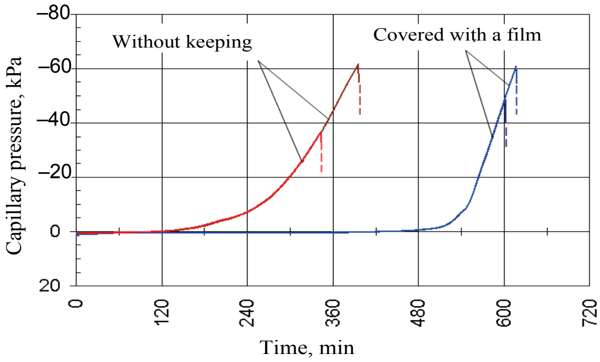
*Fig. 2.* Capillary pressure (experimental data) [10–13]

*Table 4*

**The factors, influencing capillary pressure [14–16]**

| Contributing factor | Dependence of capillary pressure |
|---|---|
| 1. Type of cement. Ashes ablation 20 °C, 45 % relative moisture |  |

*Break-over, table 4*

| Contributing factor | Dependence of capillary pressure |
|---|---|
| 2. Keeping conditions<br>a) Loss of moisture from a surface |  |
| b) Capillary pressure in time in a concrete sample at constant replenishment of amount of the evaporated water |  |
| c) Capillary pressure in time in a concrete sample at normal and moist curing and without him |  |

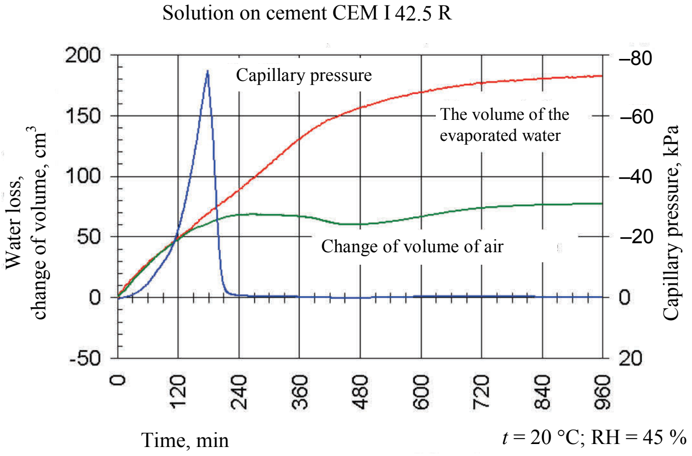


*Fig. 3.* Capillary pressure and shrinkage

## General provisions of calculation

As a theoretical basis of a method, physical ideas of the mechanism of an increment of volume of cracks in the model of concrete, presented as two-level structure are accepted: a matrix of the hardening cement stone with inclusions and emptiness of various form (crack) as result of the influences changing stress-deformed state in a point and volume.

The main criterion of a method is the generalized total parameter of crack resistance of $R$ $K_c = \sqrt{(K_{\mathrm{I}C}^2 + K_{\mathrm{II}C}^2)}$ [or $K_c(\tau)$], calculated on the basis of model schemes of development, association, localization of system of cracks, their classification by types and relative quantity in volume at the initial concentration, increasing to critical, that is caused by physical processes of change of temperature, a condition of water and physical and chemical processes of accumulation of substances of new growths.

At theoretical justification of a method of calculation of shrinkable crack resistance of concrete with use of approaches of mechanics of destruction by the generalized criterion the following assumptions are accepted.

1. Concrete is considered as the elastic quasi-homogeneous two-component medium consisting from: a) constructive part: matrixes – a cement stone with structural elements of crushed stone, sand; b) destructive part: emptiness: capillaries cracks and pores (cavities with initial cracks in walls). Initial physic-mechanical properties of concrete (constructive) are estimated by strength and deformativny characteristics of $R_b$, $R_{bt}$, $E_b$ and parameters of fracture mechanics $K_i$, $G_i$, $J_i$.

2. Emptiness in a matrix and contact zones are presented by the coordinated five-level system in a form and the sizes, multiple to diameter, under influences on reaching the critical sizes, passing from level into the following level according to the scheme: stabilization of the sizes – accumulation delocalization – critical concentration in single volume – transition to the following level.

3. Process of formation and the movement of cracks is considered as result of not power influences on the basis of the principles of the theory of cracks from a condition, that in top of each crack of the level in the initial volume of concrete, there are fields of deformations and tension creating schemes of a normal separation and shift. The arising condition is estimated by the corresponding amount of fracture energy $G_{ij}$ and stress intensity factor $K_{ij} = \sqrt{G_{ij} E_{ij}}$.

4. As the generalized constant of concrete crack resistance in pores, its resistance to formation, accumulation in volumes of micro cracks and to formation of trunk cracks of critical values the $K_{cij}(\tau)$ parameter, as the algebraic amount of critical $K_{ij}$ values in all system of all levels of the cracks, emptiness, filling canonical volume to critical concentration is accepted.

5. External temperature, moist long influences create fields of tension in tops of emptiness – cracks, which assessment is considered by parameter $D$ with application of provisions of the theory of aging of concrete:

$$K_{ic}(\tau) = K_{ic}(\tau_o) D. \tag{1}$$

6. Concrete destruction processes by cracks are considered, as the deformed state generalized intense in some initial volume, having the physical features inherent in a composite with strength and deformativny properties $R_b$, $R_{bt}$, $\nu$. Features of physical processes of moving micro and macro cracks in the studied volume are reliable and proved by experimental data by definition of $l_{crc}$, $G_i$, $J_i$, $K_{\mathrm{I}}$ and $K_{\mathrm{II}}$ on samples cubes (prisms) section of 100×100 mm in size with an optimum diameter of large inclusions no more than 15 mm.

Deformativny and strength properties in the single volume of concrete of any structure are provided with the system of active and reactive forces in structure:

$$\sum N_{act} - \sum N_{react} = R_i. \tag{2}$$

The change of external conditions, temperatures, humidity, pressure in defects of structure of $P$, $C$, $C$ filled with liquid, steam, ice arise forces, the sizes and amount of defects, quantity and property of structural ties change, that influences the level of initial properties $R_i$, $E_j$ and levels of their measured limits.

## Theoretical justifications and analytical solutions of tension and crack resistance of concrete on the basis of the generalized criterion

Let some elementary volume of a cement stone include a quantity of emptiness – the capillaries containing a certain amount of free water, depending on external conditions. Then the capillary model (the concentrator of tension, initiating emergence of micro cracks) for which walls some forces, caused by the water, which is contained in its volume are made can be presented in the form: (fig. 4, where $l_c$ – capillary length; the size $b_c$ depends on humidity of actually cement stone; $a_c$ – diameter of emptiness – a capillary).

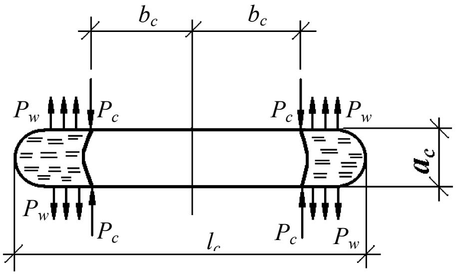


*Fig. 4.* Model of the capillary filled with water

Model parameters in characteristic points of $t$, $W$, $P$ charts of a state will be the following characteristics: $l_c$ and $a_c$ – the initial extent of emptiness – a capillary; $W$ – humidity and $t$ – temperature of a cement stone.

*State 1.* Condition: $t$ = const; $W \neq$ const; $P \neq$ const.

We will determine capillary forces by a formula

$$P_c = \pi \sigma a_c \cos\theta, \tag{3}$$

where σ – a superficial tension of liquid; θ – the angle of wetting or a regional corner on border "liquid – a capillary wall".

Proceeding from the analysis of the value σ which at a critical temperature addresses in zero it is possible to write down:

$$\sigma = \sigma_0\left(1 - t/t_k\right), \tag{4}$$

where $t_k = 370^{\circ}$ (for water); $\sigma_0 = 0{,}076$ N/м ($t = 0$).

Then force, applied to the coast of a capillary, will be defined from

$$P_c = \sigma_0 \pi a_c \cos\theta\left(1 - t/370\right). \tag{5}$$

Points of application of forces of $P_c$ depend on $W$. Considering an increment of an amount of water in a capillary due to change of humidity

$$b_c = l_c/2\left(1 - W/100\right). \tag{6}$$

At action on the top and lower coast of a crack in the points, remote from the center of a crack on distance of $b$, equal of the normal concentrated forces $P$ (but opposite in the direction) (fig. 5), the stress of intensity factor (SIF) at a normal separation of $K_{\mathrm{I}}$ is determined by Earvin's formula [17]

$$K_{\mathrm{I}} = 2P\sqrt{l}\Big/\sqrt{\pi\left(l^2 - b^2\right)}. \tag{7}$$

In the accepted designations the formula for flat tension has an appearance

$$K_{\mathrm{I}} = 2P_C\sqrt{l_c/2}\Big/\sqrt{\pi\left(\sqrt{l_c^2/4 - b_c^2}\right)} \tag{8}$$

and stress intensity factor from action of capillary forces

$$K_{\mathrm{I}} = 2P_C\sqrt{l_c/2}\Big/\sqrt{\pi\left(\sqrt{l_c^2/4 - b_c^2}\right)}\,g_c. \tag{9}$$

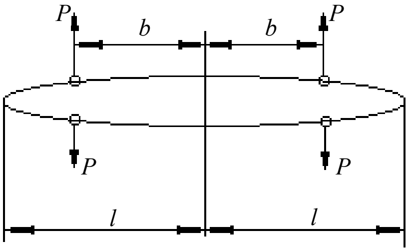


*Fig. 5.* Action on coast of a crack of several normal concentrated forces

In view of (5) and (6), we have

$$K_{\mathrm{I}} = 2\pi a_c \cos\theta \times$$
$$\times \sigma_0\left(1 - t/t_k\right)\sqrt{l_c/2}\Big/ g_c\sqrt{\pi\left(l_c^2/4 - b_c^2\right)} =$$
$$= 4\sqrt{\pi}/\sqrt{2}\, a_c \cos\theta \times$$
$$\times \sigma_0\left(1 - t/t_k\right)\Big/ g_c\sqrt{l_c\left[1 - \left(1 - W/100\right)^2\right]}.$$

Thus, the stress intensity factor at a normal separation from capillary forces is defined by the geometrical sizes of a capillary $a_c$, $l_c$, its filling with moisture $W$ and the angle of wetting of θ, a superficial tension at 0 °C $\sigma_0$ and temperature, distance between $g_c$ capillaries.

If development of a capillary in length doesn't happen, then the size of change of width (radius) of a capillary is defined

$$a_c^p = \left(2\pi / g_c E_{cs}\right) P_c / n \times \tag{10}$$
$$\times \left\{ \left[ l_c / 2 - \sqrt{\left(l_c/2\right)^2 - b_c^2} \right] \Big/ \left[ l_c/2 + \sqrt{\left(\left(l_c/2\right)^2 - b_c^2\right)} \right] \right\},$$

where $g_c$ – the distance between two next capillaries, which is function of porosity ($W/C$); $E_{cs}$ – the module of elasticity of a cement stone

With change the expert at invariable value $W$ also points of application of forces of $P_c$ (fig. 6) are displaced: originally $P_{c1}$, then $P_{c2}$.

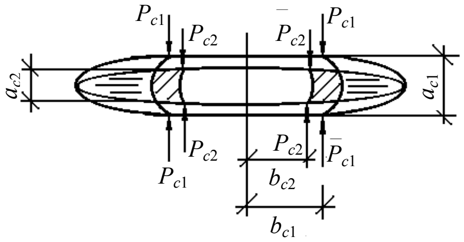


*Fig. 6.* Change of points of application of capillary forces at reduction of diameter of a capillary

Considering, that water volume in a capillary of $V = (\pi a^2{}_{c1}/4)(l_c - 2b_{c1})$ invariable, we will receive

$$b_{c2} = \left(2a_{c1}^2 b_{c1} + a_{c2}^2 l_c - a_{c1}^2 l_c\right) \Big/ 2\,a_{c2}^2,$$

where $a_{c2} = a_{c1} - a^p{}_c$.

If the humidity of a cement stone changes in the course of shrinkage, then

$$b_{c2} = \left[ l_c \left( 2a_{c1}^2 \left(1 - W/100\right)/2 + a_{c2}^2 - a_{c1}^2 \right) \right] \Big/ 2a_{c2}^2 \mp$$
$$\mp \left(l_c/2\right)\left(1 - \Delta W/100\right) =$$
$$= l_c/2 \left[ \left( 1 - \left(a_{c1}/a_{c2}\right)^2 \left(W/100\right) \pm \left(1 - \Delta W/100\right) \right) \right],$$

where $\Delta W$ – change of humidity: the sign "–" at increase in W, the sign "+" at reduction.

Then intensity of tension in capillary top

$$K_{\mathrm{I}} = 2\pi a_{c2} \cos\theta \cdot \sigma_0 \left(1 - t/t_k\right) \times$$
$$\times \sqrt{l_c/2} \Big/ g_c \sqrt{\pi \left( \sqrt{l_c^2/4 - b_c^2} \right)}.$$

Shrinkage deformation, if to take into account, that capillaries (micro cracks) are evenly distributed on concrete volume, is defined from

$$\varepsilon_{sh} = P_c G_{\mathrm{I}c} \Big/ \left( l_c K_{\mathrm{I}c}^2 a_c \right), \tag{11}$$

where $G_{\mathrm{I}c}$ – energy of destruction of a cement stone.

In the direction, parallel to action of forces of $P_c$, the main shifting tension causing in tops of a capillary of deformation of cross shift described by stress intensity factor of $K_{\mathrm{II}}$, which size is attached to a capillary

$$K_{\mathrm{II}} = \tau \sqrt{\pi l_c}, \tag{12}$$

where $\tau$ – main tangent tension.

We will define them, in view of that capillaries are evenly distributed on the area of concrete. Taking into account (11) and (12)

$$K_{\mathrm{II}} = P_c \sqrt{\pi l_c} \Big/ \left(g_c a_c\right).$$

At the time of $K_{\mathrm{II}} = K_{\mathrm{II}}^{cs}$ growth of a micro crack in length will be defined by the mechanism of cross shift.

*State 2.* Condition: $W$ = const; $t \neq$ const; $P \neq$ const.

We will consider a cement stone at the macro level. It consists of not hydrated grain and the hydrated weight which in turn consists of emptiness – pores (capillaries) and crystal system (micro level).

In the hydrated weight micro defects of two types will be observed: I – capillaries; II – the cracks of a normal separation formed because of the difference of modules of elasticity and coefficients of linear expansion of not hydrated grain and the hydrated weight. Then the general resilience of a cement stone to development of temperature cracks in terms of stress intensity factors are defined from:

$$K_{\mathrm{I},t}^{cs} = K_{\mathrm{I},t}^{\mathrm{I}} + K_{\mathrm{I},t}^{\mathrm{II}};$$

$$K_{\mathrm{II},t}^{cs} = K_{\mathrm{II},t}^{\mathrm{I}} + K_{\mathrm{II},t}^{\mathrm{II}},$$

where $K_{\mathrm{I},t}^{\mathrm{I}}$, $K_{\mathrm{II},t}^{\mathrm{I}}$ – the stress intensity factors in top of capillaries, caused by the intra capillary pressure of water; $K_{\mathrm{I},t}^{\mathrm{II}}$, $K_{\mathrm{II},t}^{\mathrm{II}}$ – the same in tops of cracks like II.

Then at the time of development of microcracks and their combining in trunk macrocracks

$$K_{\mathrm{I}c,t}^{cs} = K_{\mathrm{I}c,t}^{\mathrm{I}} + K_{\mathrm{I}c,t}^{\mathrm{II}};$$

$$K_{\mathrm{II}c,t}^{cs} = K_{\mathrm{II}c,t}^{\mathrm{I}} + K_{\mathrm{II}c,t}^{\mathrm{II}}.$$

We will consider capillary micro defects in the temperature range: 1 (water).

*Range 1 (water).* The capillary is affected by the system of forces shown in fig. 7. Forces of $P_c$ are defined by capillary pressure; $P_w$ – expansion of water at temperature increase; $P_{cs}$ – expansion of crystal system; $\tau$ – the tangent tension, arising from action of forces in the capillaries focused parallel to forces of $P$.

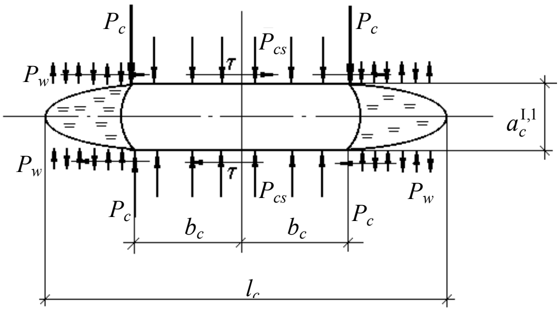


*Fig. 7.* The system of forces operating on the capillary filled with water

Thus

$$K^{\mathrm{I},1}_{\mathrm{I},t} = K^{\mathrm{I},1,C}_{\mathrm{I},t} - K^{\mathrm{I},1,W}_{\mathrm{I},t} + K^{\mathrm{I},1,CS}_{\mathrm{I},t},$$

or at the time of local destruction

$$K^{\mathrm{I},1}_{\mathrm{I},C,t} = K^{\mathrm{I},1,C}_{\mathrm{I},C,t} - K^{\mathrm{I},1,W}_{\mathrm{I},C,t} + K^{\mathrm{I},1,CS}_{\mathrm{I},C,t}.$$

Then

$$K^{\mathrm{I},1,C}_{\mathrm{I},t} = \left(4\sqrt{\pi}/\sqrt{2}\right) a_c \cos\theta \times$$
$$\times \sigma_0 \left(1 - t/t_{к}\right)/g_c \sqrt{l_c\left[1-\left(1-W/100\right)^2\right]}.$$

The size $P_w$ will be defined from

$$P_w = \alpha_{t,w} \Delta t\, E_w$$

and the size $\alpha_{t,w}$ on the basis of the analysis of skilled data

$$\alpha_{t,w} = 0{,}000067 + 0{,}0000076t.$$

At action of constant normal loading by intensity of $\sigma$ on the symmetric trailer sites of a crack adjoining tops of a crack (fig. 8) according to G. Sih's decision [18], SIF at a normal separation is determined by a formula

$$K_{\mathrm{I}} = \sigma\sqrt{\pi l}\left[1-\left(2/\pi\right)\arcsin\left(a/l\right)\right].$$

Then SIF from temperature expansion of the water closed by capillary forces in capillary tops

$$K^{\mathrm{I},1,W}_{\mathrm{I},t} = \alpha_{t,w}\Delta T\, E_w\sqrt{\pi l_c/2}\left[1-\left(2/\pi\right)\arcsin\left(2b_c/l_c\right)\right]. \quad (13)$$

The size $P_{cs}$ will be defined from

$$P_{cs} = \alpha_{t,cs}\Delta T\, E_{cs}.$$

At action on coast of a crack of a constant of normal ($\sigma$) and the loading (fig. 9) of KIN moving ($\tau$ at a normal separation and cross shift is determined by G. P. Cherepanov [19] and V. V. Panasyuk's formulas [20]:

$$K_{\mathrm{I}} = \sigma\sqrt{\pi l};$$
$$K_{\mathrm{II}} = \tau\sqrt{\pi l}.$$

Then SIF from temperature expansion of crystal system is calculated from expression

$$K^{\mathrm{I},1,cs}_{\mathrm{I},t} = \alpha_{t,cs}\Delta T\, E_{cs}\sqrt{\pi l_c/2}. \quad (14)$$

If $K^{\mathrm{I},1,C}_{\mathrm{I}C,t} + K^{\mathrm{I},1,cs}_{\mathrm{I}C,t} \succ K^{\mathrm{I},1,W}_{\mathrm{I}C,t}$, then size $a_c^{\mathrm{I},1}$ decreases and vice versa. We will determine width of disclosure (radius) of a capillary from

$$a_c^{\mathrm{I},1} = a_c^b + a_c^{cs} - a_c^c, \quad (15)$$

where all entering (15) parameters are determined by a formula (10).

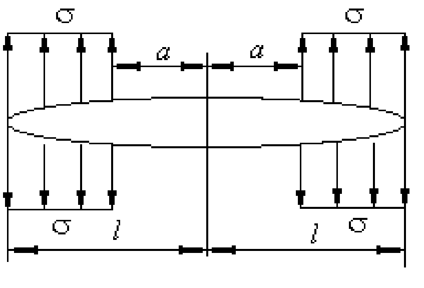


*Fig. 8.* Action of continuous normal forces on symmetric trailer sites of cracks

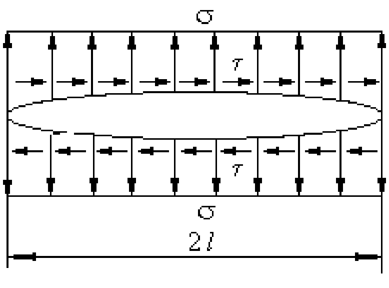


*Fig. 9.* Action of constant loading on coast of a crack

Size $K_{\mathrm{II},t}{}^{\mathrm{I},1}$ is defined on

$$K_{\mathrm{II},t}^{\mathrm{I},1}=\left[\sigma_0\pi a_c\cos\theta\left(1-t/t_k\right)+\alpha_{t,cs}\Delta tE_{cs}l_cg_c-\right.$$
$$\left.-2\alpha_{t,w}\Delta tE_wg_c\left(l_c/2-b_c\right)\right]\sqrt{\pi l_c}\Big/\sqrt{2}\,g_c^2. \tag{16}$$

Depending on orientation of the closed cracks or capillaries, they can be filled with liquid asymmetrically, then the intensity of tension in tops of defect won't be identical. Such capillary is affected by the system of forces shown in fig. 10. At the same time

$$K_{\mathrm{I},t}^{\mathrm{I},1}+K_{\mathrm{I},t}^{\mathrm{I},1,C,m}-K_{\mathrm{I},t}^{\mathrm{I},1,W,m}+K_{\mathrm{I},t}^{\mathrm{I},1,cs}, \tag{17}$$

where $K_{\mathrm{I},t}^{\mathrm{I},1,C,m}=\left(K_{\mathrm{I},t}^{\mathrm{I},1,C,A}+K_{\mathrm{I},t}^{\mathrm{I},1,C,B}\right)\Big/2$; $K_{\mathrm{I},t}^{\mathrm{I},1,C,A}$ – stress intensity factor in a point $A$ from action of forces $P_c$; $K_{\mathrm{I},t}^{\mathrm{I},1,C,B}$ – the same, in a point $B$; $K_{\mathrm{I},t}^{\mathrm{I},1,W,m}=\left(K_{\mathrm{I},t}^{\mathrm{I},1,W,A}+K_{\mathrm{I},t}^{\mathrm{I},1,W,B}\right)\Big/2$; $K_{\mathrm{I},t}^{\mathrm{I},1,W,A}$ – SIF in a point $A$ from action of forces $P_w$; $K_{\mathrm{I},t}^{\mathrm{I},1,W,B}$ – the same, in a point $B$.

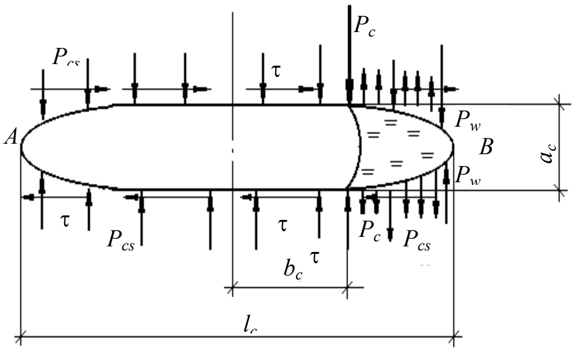


*Fig. 10.* The system of forces operating on the capillary which is asymmetrically filled with water

When on the top and lower coast of a crack in the point remote from the center of a crack on $b$ distance, the concentrated forces (fig. 11), SIF, normal, opposite in the direction, according to decisions V. V. Panasyuk, M. P. Savruk, A. P. Datsyshin, J. Sih, G. Libovits are applied, P. Paris, J. Irvin, G. P. Cherepanov [21–23, 19, 17, 18, 24] are defined from expression

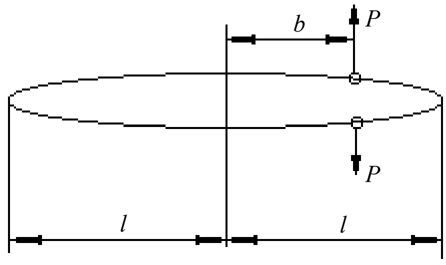


*Fig. 11.* Action on coast of a crack of the concentrated forces

$$K_{\mathrm{I}}^{\pm}-iK_{\mathrm{II}}^{\pm}=\left(1\big/\sqrt{\pi l}\right)\left[\left(P-iQ\right)\sqrt{\left(l\pm b\right)/\left(l\mp b\right)}\pm\right.$$
$$\left.\pm Ml\Big/\left(\left(l\mp b\right)\sqrt{l^2-b^2}\right)\right],$$

here and further the sizes $K_{\mathrm{I}}^{\pm}$ and $K_{\mathrm{II}}^{\pm}$ with the lower sign belong to the left top of a crack ($x=-l$), and with top – to right ($x=l$).

Then stress intensive factor from capillary forces in points $A$ and $B$ respectively:

$$K_{\mathrm{I},t}^{\mathrm{I},1,C,A}=P_c\sqrt{\left(l_c/2-b_c\right)/\left(l_c/2+b_c\right)}\Big/\sqrt{\pi l_c/2};$$

$$K_{\mathrm{I},t}^{\mathrm{I},1,C,B}=P_c\sqrt{\left(l_c/2+b_c\right)/\left(l_c/2-b_c\right)}\Big/\sqrt{\pi l_c/2};$$

$$K_{\mathrm{I},t}^{\mathrm{I},1,C,m}=\left(K_{\mathrm{I},t}^{\mathrm{I},1,C,A}+K_{\mathrm{I},t}^{\mathrm{I},1,C,B}\right)\Big/2=$$
$$=P_c\sqrt{2}\Big/\sqrt{\pi\left(l_c^2-4b^2\right)}.$$

When on coast of cracks on site $b\le x\le c$ are enclosed constant normal (σ) and shifting (τ) of effort (fig. 12), using J. Sih and P. Paris's decision [24], SIF equal

$$K_{\mathrm{I}}^{\pm}-iK_{\mathrm{II}}^{\pm}=\left(\sigma-i\tau\sqrt{l/\pi}\ge\left[\arcsin\left(c/l\right)-\right.\right.$$
$$\left.\left.-\arcsin\left(b/l\right)\mp\sqrt{1-\left(c/l\right)^2}\pm\sqrt{1-\left(b/l\right)^2}\right]\right). \tag{18}$$

From where SIF from action of forces of $P_w$ in points $A$ and $B$ a capillary will be:

$$K_{\mathrm{I},t}^{\mathrm{I},1,W,A}=\alpha_{t,w}\Delta tE_w\sqrt{l_c/2\pi}\times$$
$$\times\left[\pi/2-\arcsin\left(2b_c/l_c\right)-\sqrt{1-\left(2b_c/l_c\right)^2}\right];$$

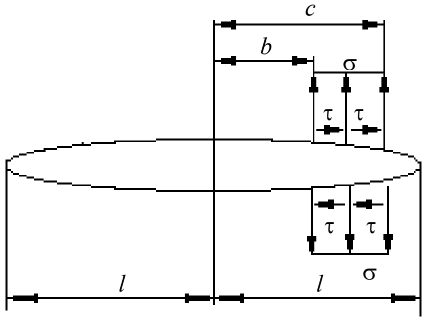


*Fig. 12.* Action of constant loading on the internal site of the top and lower coast of a crack

$$K_{\mathrm{I},t}^{\mathrm{I},1,W,B} = \alpha_{t,w}\Delta t E_w\sqrt{l_c/2\pi}\times$$
$$\times\left[\pi/2-\arcsin\left(2b_c/l_c\right)+\sqrt{1-\left(2b_c/l_c\right)^2}\right];$$

$$K_{\mathrm{I},t}^{\mathrm{I},1,W,m} = \alpha_{t,w}\Delta t E_w\sqrt{l_c/2\pi}\left[\pi/2-\arcsin\left(2b_c/l_c\right)\right].$$

The stress intensity factor in each of tops of the capillary which is asymmetrically filled with water can be defined from:

$$K_{\mathrm{I},t}^{\mathrm{I},1,A} = K_{\mathrm{I},t}^{\mathrm{I},1,C,A} - K_{\mathrm{I},t}^{\mathrm{I},1,W,A} + l/2\,K_{\mathrm{I},t}^{\mathrm{I},1,cs};$$

$$K_{\mathrm{I},t}^{\mathrm{I},1,B} = K_{\mathrm{I},t}^{\mathrm{I},1,C,B} - K_{\mathrm{I},t}^{\mathrm{I},1,W,B} + l/2\,K_{\mathrm{I},t}^{\mathrm{I},1,cs}.$$

We will consider the regional cracks or not closed capillaries (index II) coming to a surface (side) of a sample. In the first temperature range the crack (time) is affected by the system of forces shown in fig. 13, at the same time

$$K_{\mathrm{I},t}^{\mathrm{II},1} = K_{\mathrm{I},t}^{\mathrm{II},1,C} - K_{\mathrm{I},t}^{\mathrm{II},1,W} + K_{\mathrm{I},t}^{\mathrm{II},1,cs}. \qquad (19)$$

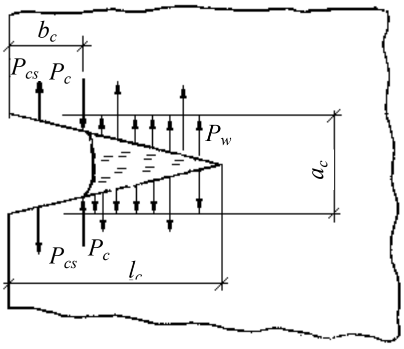


*Fig. 13.* The system of forces operating on not closed capillary in the first temperature range

Let the return be attached to opposite coast of a crack at b distance from the region of the half-plane equal in size, but in the direction the normal and tangent concentrated forces (fig. 14). Then from the decision, received by V. V. Panasyuk, M. P. Savruk, A. P. Datsyshin [21–23] by means of special approximation of the singular integrated equation follows

$$K_{\mathrm{I}} - iK_{\mathrm{II}} = 2\left(P-iQ\right)\sqrt{c/2\pi l}\Big/\sqrt{1-\left(b/l\right)^c},$$

where $c = 2\pi^2\big/\left(\pi^2-4\right)$.

Then intensity of tension in top of regional defect from action of capillary forces

$$K_{\mathrm{I},t}^{\mathrm{II},1,C} = 2P_c\sqrt{c/2\pi l_c}\Big/\sqrt{1-\left(b_c/l_c\right)^c} =$$
$$= \sqrt{2c}\,P_c\Big/\sqrt{\pi l_c\left[1-\left(b_c/l_c\right)^c\right]}. \qquad (20)$$

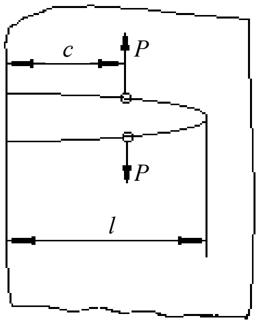


*Fig. 14.* The half-plane with a regional crack at action in any points of its coast of the concentrated forces

When on coast of a crack piece-vise and constant loading (fig. 15) is set, and the region of the half-plane is free from tension, the numerical solution of the integrated equations on the basis of which by method of interpolation of R. Hartranft and J. Sih have constructed analytical expression for SIF [25, 18] was used

$$K_{\mathrm{I}} - iK_{\mathrm{II}} =$$
$$= 2\left(P-i\tau\right)\sqrt{\pi l}\left(2/\pi\right)\arccos\left(b/l\right)\left[1+f\left(b/l\right)\right].$$

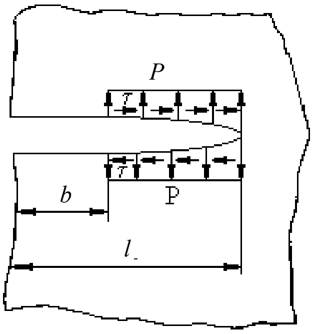


*Fig. 15.* Action of piecewise and constant loading on coast of a regional crack in the half-plane

Values of the *f*(*b*/*l*) function are given by G. Sih [18] and can be approximated by expression *f*(*b*/*l*) = 0,1215(1 – *b*/*l*).

Intensity of tension in top of regional defect from linear expansion porovoy waters

$$K_{\mathrm{I},t}^{\mathrm{II},1,W} = P_w\sqrt{\pi l_c}\left(2/\pi\right)\arccos\left(b_c/l_c\right)\times$$
$$\times\left[1+f\left(b_c/l_c\right)\right] = 2P_w\sqrt{l_c/\pi}\times \qquad (21)$$
$$\times\, ABC\arccos\left(b_c/l_c\right)\left[1+0{,}1215\left(1-b_c/l_c\right)\right].$$

The stress intensity factor caused by expansion of crystal system at increase in temperature is defined from expression

$$K_{\mathrm{I},t}^{\mathrm{II},1,cs} = P_{cs}\sqrt{\pi l_c}\,(2/\pi)\arccos(o)\left[1+f(o)\right] =$$
$$= \alpha_{t,cs}\Delta t E_{cs}\sqrt{\pi l_c}\,(2/\pi)1{,}5708\left[1+0{,}1215\right] = \qquad (22)$$
$$= 3{,}523\alpha_{t,cs}\Delta t E_{cs}\sqrt{l_c/\pi}.$$

## CONCLUSIONS

1. As a theoretical basis of a method, physical ideas of the mechanism of an increment of volume of cracks in concrete shrinkage model are accepted.
2. The main criterion of a method is the generalized total parameter of crack resistance $K_c$.
3. Modern ideas of the mechanism of moist shrinkage, experimental data of the value of capillary pressure (70 kPa in 180 min) allow to execute analytical decisions for assessment of tension and crack resistance of concrete at early age on the basis of the generalized criterion in terms of coefficients of stress intensity factors.
4. The developed algorithm of calculation of crack resistance at shrinkage allows to consider the factors influencing capillary pressure: a type of cement, existence of modifiers and mineral additives, conditions of keeping of concrete (superficial wetting, completion of the evaporated water, normal and moist curing).

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